Supercondcting Qubits

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Superconducting qubits are electrical circuits based on the Josephson tunnel junctions and have the ability to condense electrons into Cooper pairs forming a single superfulid, thus enabling movement throughout the metal lattice without resistance. Due to the non-dissipative and non-linear property of the Josephson junction it is an ideal circuit element at low temperature conditions. In addition, the use of multiple junctions in circuits has eliminated extrinsic electromagnetic perturbations, maintaining qubit coherence.

Implementation of quantum computation algorithms requires a controllable quantum network. While some qubits, such as the trapped ions, are constructed from microscopic degrees of freedom superconducting qubits are constructed from macroscopic degrees of freedom relying on the electrodynamic modes of electrical components. Since the circuit relies heavily on the absence of energy dissipation, metallic components of the circuit needs to be made out of a material with zero resistance at low temperatures, such as aluminum or niobium [3]. Low noise filtering can be achieved by the low temperature of the circuit and requires that the thermal fluctuations $k_B T$ is less than that of the quantum energy $\hbar\omega_{01}$ (transition between 0 and 1 states).

Nakamura and his coworkers at NEC experimentally observed coherent quantum oscillation in a single Cooper pair box in 1999 [1]. The formation of quantized Cooper pairs on a superconducting island was of significance since it is the foundation of the charge qubit. A charge qubit is another form of a superconducting qubit with basis states corresponding to charge states, which is determined by the number of Cooper pairs that have tunneled across the Josephson junction.

At zero temperature the electrons are condensed into Cooper pairs through a small attraction between electrons in metal, the subsequent formation of a superfluid enables the movement throughout the metal lattice without any resistance. Therefore, it is the Cooper pair that is responsible for the superconductivity of the circuit. Movement of the superfluid is determined by a single wavefunction since all pairs fall into the ground state, its amplitude (K) determines the macroscopic number of Cooper pairs formed, and the phase of the wavefunction ($\theta(r)$) is related to the supercurrent and magnetic fields that may also be present.

The number of Cooper pairs can be interpreted in terms of a Cooper pair density $\rho(r) = |\Psi(r)|^2$.

$$\Psi(r) = (\rho(r))^{1/2} e^{i\theta(r)}$$

Where the phase of the wavefunction at point r is $\theta(r)$. Cooper pairs are held by a weak interaction of the order 10^-3eV and can be broken up with sufficient energy, which is referred to as the gap energy. The gap energy is the energy difference between the ground and the first excited states. In Aluminum, the energy gap corresponds to a frequency of 90 GHz at 20 mK [5].

Manipulations of qubits are carried out through the exposure of qubits to periodic electric or magnetic field, resulting in a phenomenon referred to as Rabi oscillations [7]. The state transition is induced by applying microwave signals to the currents with a frequency resonant with the qubit level of separation [2]. In linear oscillators, the energy difference between successive energy levels is $\hbar\omega$ and the equal energy spacing of $\hbar\omega$ is due to the linearity of the oscillator. Therefore, in order to produce Rabi oscillations between the ground and first excited level a nonlinear and nondissipative device such as the Josephson junction must be used. A transition frequency ranges from 5-20 GHz and is related to the classical plasma oscillation frequency by the relation $\omega_{01} \simeq 0.95\ \omega_p$, where ω_{01} is the transition frequency between states |0> and |1> and ω_p is the plasma frequency. The plasma oscillation frequency is related to the circuit components by

$$\omega_p = (1/(L_{J0}C_J))(1 - (I_J/I_0)^2)^{1/4} \qquad (2)$$

where L_{J0} is the inductor, C_J is the capacitor, I_J is the Josephson current, and I_0 is the critical current. The transition frequency can be controlled by applying an external magnetic flux to the Josephson junctions [6].

Josephson junction is an electrical component that incorporates a thin layer of insulator, usually a few nanometers in thickness, which is sandwiched between two metals that become a superconductor below 1.2K [5]. The insulator is thin enough to allow electrons to travel across it and is referred to a quantum process known as tunneling. As a voltage bias V is applied to the junction, the right and left side of the superconducting junction given by the equation given above can be described a two-level system with a coupled wavefunction due to tunneling:

$$i\hbar(d\Psi_1/dt) = ½\ q_cV\ \Psi_1 + K\ \Psi_2,$$

$$i\hbar(d\Psi_2/dt) = -½\ q_cV\ \Psi_2 + K\ \Psi_1.$$

The wavefunctions on the right and left side of the junction is given by Ψ_1 and Ψ_2, respectively. K is the amplitude of Ψ and q_c is the charge of the Cooper pair, which is twice the value of an electric charge. The Josephson current I_J across the junction is

$$I_J(t) = I_0 \sin[2\pi\Phi(t)/\Phi_0]$$

The periodic flux dependence of current arises from discreet Cooper pair tunneling.

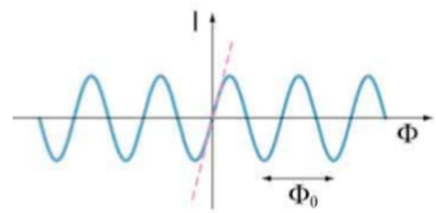

Figure 1. Josephson junction current-flux relationship. The solid line represents nonlinear and non-dissipative electrical element and the dashed line represents linear inductance equal to the junction effective inductance [3].

The Josephson current is governed by the universal quantum constant Φ_0, the magnetic flux $\Phi(t)$, and the a critical current I_0 is the maximum current that can produce zero resistivity in the metal. $2\pi\Phi(t)/\Phi_0 = \delta$ is referred to as the gauge-invariant phase difference, which occurs across the junction and is related to the phase difference between two superconducting ground state condensates on either side of the junction by

$$\theta = \delta \bmod 2\pi.$$

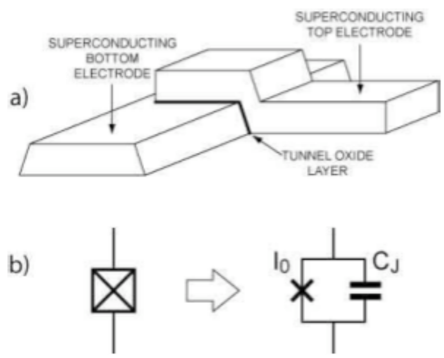

Figure 2. Josephson Junction Representation
a) Josephson tunnel junction made with two superconducting thin films; b) Schematic representation of a Josephson tunnel junction.

The Josephson tunnel junction behaves like a pure nonlinear inductor in parallel with a capacitor at low temperature, voltage, and applied frequency. This equivalent model allows complex quantum circuits to be analyzed by conventional circuit theory.

Another parameter describing the Josephson nonlinear inductance is given by the phase-dependant equation

$$L_{J0} = \Phi 0 / 2\pi$$

$$L_J(\delta) = (\partial I / \partial \Phi)^{-1} = L_{J0} / \cos \delta$$

Due to the $1/\cos \delta$ term, the inductance is highly nonlinear and corresponds to the nonlinearity observed in the Josephson junction [3].

Designing the superconducting qubit requires two types of energy, the Josephson energy E_J and the Coulomb charging energy E_c. The Josephson energy is a measure of the strength of coupling across the junction, whereas the Coulomb charge energy is the energy needed to increase the charge of the junction by 2e [5]. Cooper pairs and phase difference can be manipulated by varying the magnitude of E_c and E_J. If the value of E_C is larger than E_J, the number of Cooper pairs remains fixed and the tunneling of the pair leads to the creation of the charge qubit as it undergoes state transitions. If E_J is larger than E_C, the total phase around the closed loop of the circuit is a multiple of 2π [5].

The drawback of superconducting qubits is short lifetime of coherence. In networks that exploit microscopic degrees of freedom such as the trapped ions and atoms, the coherence time is approximately 1s, whereas the superconducing qubit has coherence times ranging from 500ns to 4μs [5]. Since coherence is essential to prevent leakage of information carried by the qubit states, it is essential to create superconducting quantum network that increases qubit coherence times.

Decoherence is mostly caused by materials in the circuit, such as defects, rather than external influence on the circuit. A geometrical representation of the qubit relaxation and decoherence can be made using a Bloch sphere. Relaxation corresponds to the movement of the tip of the unit vector, which represents the quantum state, in the latitude direction. Decoherence is represented by the diffusion of the vector in the longitudinal direction. Both decoherence and relaxation rates are directly proportional to the total spectral density in the (6) qubit Larmor frequency

$$\omega = \gamma B, \ \gamma = -eg/2m$$

where ω is the Larmor frequency, γ is the gyromagnetic ratio, B is the magnetic field, and g is the g-factor [3].

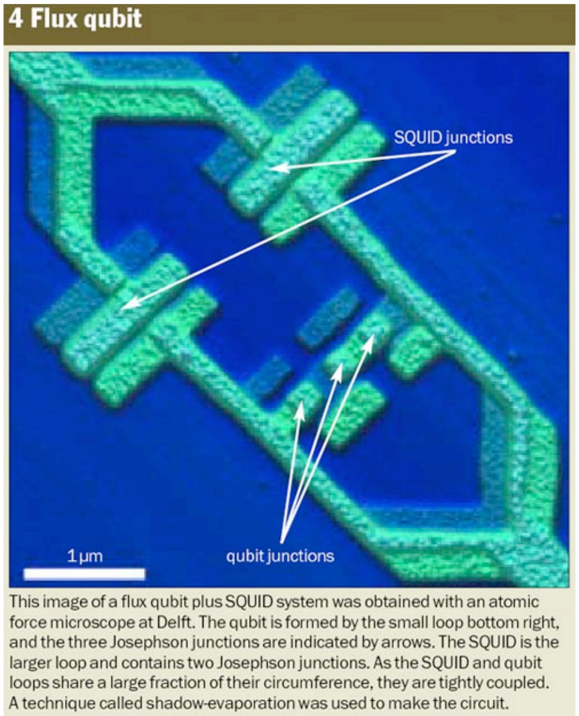

Figure 4. Flux Qubit Circuit [5]

A research group in Netherlands, in collaboration with MIT, focuses on the flux qubits since the magnetic noise is smaller than the electrical noise created by defects in the circuit. Three aluminum Josephson junctions are used and measurements are made using a superconducting quantum interference device (SQUID). The SQUID is a magnetic flux detector that consists of two junctions in parallel (Fig. 4). Absorption of microwave radiation was measured and the spectroscopic data indicated the formation of two new symmetric and antisymmentric superposition of states. In addition, Rabi oscillations were also performed using a new spectroscopic measurement technique on two coupled flux qubits [5].

Another experiment conducted at Yale created coherence by running a separate control wire to each qubit. The current on the control wire determined its operating frequency and the two-qubit interaction could be quickly turned on to exchange a virtual photon in the microwave resonator cavity [4]. A basic logic gate and a set of logic steps were implemented to carry out the Grover search (Fig. 5) and Deutsch-Josza quantum algorithms. The Deutsch-Josza algorithm enables the search of an unsorted database with n entries to find the answer after a number of tries on the order of $n^{1/2}$, where classically the search would require n tires.

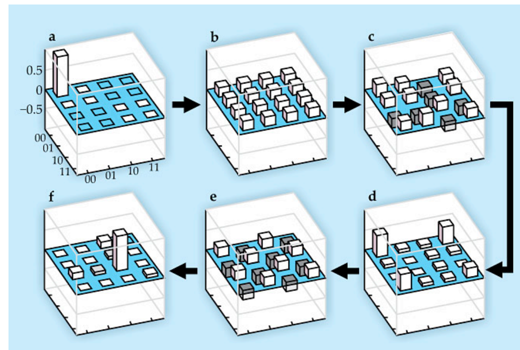

Figure 5. The Grover algorithm implemented in six steps (a - f). Each panel depicts the occupation of each state as a function of four basis states, |0,0〉, |0,1〉, |1,0〉, and |1,1〉 [4].

Quantum mechanical properties of superposition and state entanglement are necessary to achieve massive parallel processing. Through the modification of the superconducting circuit, coherence may possibly exceed the current 4μs. In addition, decoherence times must be eliminated through the improvement of tunnel barriers. Therefore, progress in fabrication technology and qubit coupling techniques is an essential component in the advancement of the superconducting quantum processor.

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